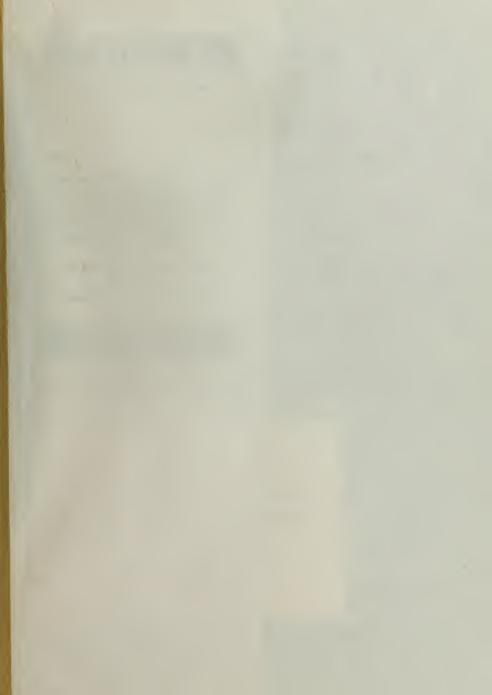
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#### ANTENNA LABORATORY

Technical Report No. 16

A Company

# THE CHARACTERISTIC IMPEDANCE OF THE

# FIN ANTENNA OF INFINITE LENGTH

by Robert L. Carrel

15 January 1957

Contract No. AF33(616)-3220 Project No. 6(7-4600) Task 40572 WRIGHT AIR DEVELOPMENT CENTER



ELECTRICAL ENGINEERING RESEARCH LABORATORY
ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS



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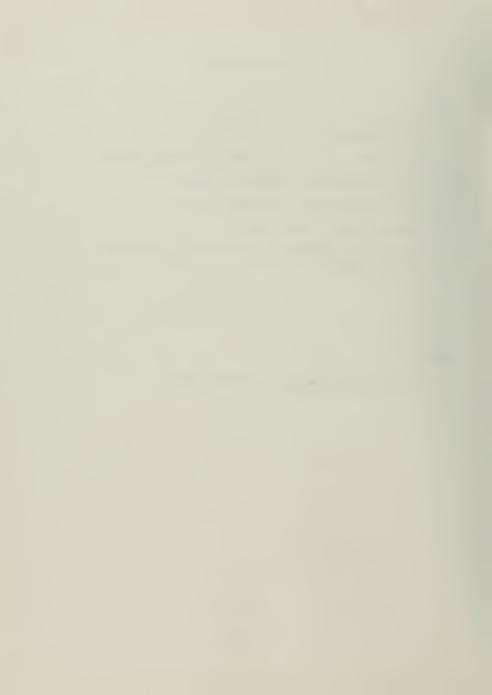
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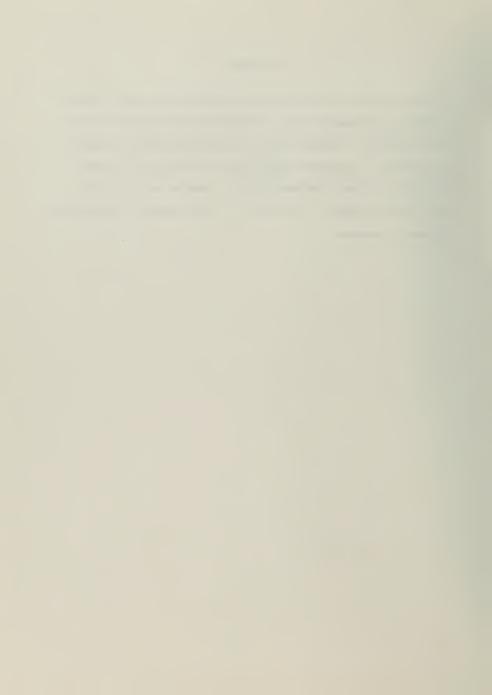
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#### ABSTRACT

Certain types of multi-conductor structures will support spherical transverse electromagnetic waves. The infinite fin antenna is one of these structures. A general method of attacking the static boundary value problem, as applied to these special structures, is outlined. The problem of finding the characteristic impedance of the infinite fin is solved completely. In addition, a short discussion of spherical TEM waves is appended.



#### ACKNOWLEDGEMENT

The author gratefully acknowledges the assistance and directive guidance of Professors V.H. Rumsey and R.H. DuHamel in the preparation of this paper.



#### 1. INTRODUCTION

The infinite fin is a member of the family of antennas whose shapes are defined entirely in terms of angles, hence their electrical characteristics are independent of frequency. See Figure 1.

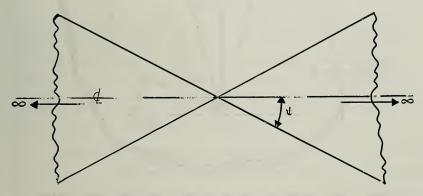


Figure | The Infinite Fin Antenna

In contrast to the infinite biconical antenna<sup>1</sup>, which is a surface of revolution, the fin antenna is infinitely thin and lies entirely in a plane.

Use of the method of images can be made to facilitate the solution of this problem. Thus we need only be concerned with one hemisphere of the spherical coordinate system. See Figure 2. The characteristic impedance of such a structure will be half of the characteristic impedance of the total structure with the image plane removed.

<sup>1.</sup> S.A. Schelkunoff and H.T. Friis, "Antennas: Theory and Practice" Wiley, 1952, pp. 104-106

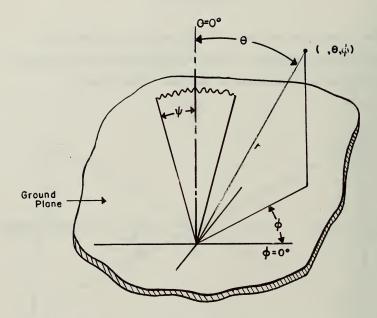


Figure 2 Position of the Fin in the Spherical Coordinate System

Since the shape of the antenna is clearly independent of r, the radius of the spherical coordinate system, only two parameters,  $\theta$  and  $\phi$ , are needed to define the structure. Also, since any spherical surface (r=constant) will intersect the antenna and its image plane in an invariant manner for any r, the problem will be shown to reduce to essentially a two-dimensional problem in electrostatics.

#### 2. FORMULATION OF THE PROBLEM

It now becomes necessary to examine the form of the solution of Maxwell's equations in spherical coordinates with the restriction that  $E_{\mathbf{r}} = 0 = H_{\mathbf{r}}$ . It can be shown that Maxwell's equations reduce to the two-dimensional Laplace' equation,

$$\sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \mathbf{T}}{\partial \theta} \right) + \frac{\partial^2 \mathbf{T}}{\partial \phi^2} = 0, \tag{1}$$

where T is a function of  $\theta$  and  $\phi$  only. The  $\underline{E}$  and  $\underline{H}$  vectors describing the field contain no radial component, and propagation is in the r direction only. Thus we conclude that the conductors support spherical transverse electromagnetic (TEM) waves, and that these waves permeate all space except that occupied by the conductors.

In addition, any solution must satisfy the boundary conditions, namely, that tangential E and normal H must vanish on the surface of the conductors. It can be shown that these boundary conditions reduce to those of electrostatics. The reader unfamiliar with this underlying development may refer to the appendix. Thus the problem is drastically reduced to a simpler one, namely, the solving of Laplace's equation in two dimensions subject to the boundary conditions of electrostatics.

The use of conjugate function theory in solving two-dimensional electrostatic potential problems has been extensive and solutions of many problems have been tabulated in the literature. It will be convenient then for us to draw a close analogy between our problem and the classical two-dimensional problem. The classical problem postulates that the equipotential surfaces must be cylindrical surfaces of constant cross section.

Mathematically speaking dV/dz = 0 in the cylindrical coordinate system, where V is the potential function. It can be seen that a given conical surface, when defined by angles, will intersect all spheres centered at the apex of the cone in a similar manner. This is analogous to the intersection of a cylindrical surface and any plane (z = constant) in the cylindrical coordinate system.

The analogy will now be complete if we can map the spherical surface onto a plane in such a way that the identity of the boundaries is preserved and that Laplace's equation remains unchanged. In other words, we are looking for a relation

$$\rho = f(\theta)$$

$$\phi_c = \phi$$
(2)

which will provide this desired mapping. (In this and the following developments  $\rho$ ,  $\phi_c$ , and z are cylindrical coordinates; r,  $\rho$ , and  $\phi$  are spherical coordinates).

We begin by examining Laplace's equation. In cylindrical coordinates

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi_c^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$
 (3)

In spherical coordinates

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 U}{\partial \phi^2} = 0. \tag{4}$$

If dV/dz = 0 and dU/dr = 0, (the initial premise of the theory), Equation 3 becomes

$$\frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho} \frac{\partial^2 V}{\partial \varphi^2} = 0.$$
 (5)

Equation 4 becomes

$$\frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{\sin \theta} \frac{\partial^2 U}{\partial \phi^2} = 0. \tag{6}$$

Substitute Equation 2 in Equation 5. Steps taken in the substitution are:

$$\frac{\partial \mathbf{V}}{\partial \rho} = \frac{\partial \mathbf{V}}{\partial \theta} \frac{\partial \theta}{\partial \rho} = \frac{1}{\mathbf{f'}(\theta)} \frac{\partial \mathbf{V}}{\partial \theta'}$$

$$\rho \frac{\partial \mathbf{V}}{\partial \rho} = \frac{\mathbf{f}(\theta)}{\mathbf{f}'(\theta)} \frac{\partial \mathbf{V}}{\partial \theta} ,$$

$$\frac{\partial}{\partial \rho} \left( \rho \frac{\partial \mathbf{V}}{\partial \rho} \right) = \frac{\partial}{\partial \theta} \left( \rho \frac{\partial \mathbf{V}}{\partial \rho} \right) \frac{\partial \theta}{\partial \rho} = \frac{1}{\mathbf{f'}(\theta)} \frac{\partial}{\partial \theta} \left[ \frac{\mathbf{f}(\theta)}{\mathbf{f'}(\theta)} \frac{\partial \mathbf{V}}{\partial \theta} \right].$$

Then Equation 5 becomes

$$\frac{1}{\mathbf{f}'(\theta)} \stackrel{\partial}{\partial \theta} \left[ \begin{array}{cc} \underline{\mathbf{f}}'(\theta) & \partial \underline{V} \\ \underline{\mathbf{f}}'(\theta) & \partial \theta \end{array} \right] + \frac{1}{\mathbf{f}(\theta)} \stackrel{\partial^2 \underline{V}}{\partial \phi^2} = 0.$$

Multiplying by  $f'(\theta)$ ,

$$\frac{\partial}{\partial \theta} \begin{bmatrix} \mathbf{f}_{\cdot}(\theta) & \partial \mathbf{V} \\ \mathbf{f}'(\theta) & \partial \theta \end{bmatrix} + \frac{\mathbf{f}'(\theta)}{\mathbf{f}_{\cdot}(\theta)} \frac{\partial^2 \mathbf{V}}{\partial \varphi^2} = 0. \tag{7}$$

Comparing 7 with 6 we see that these equations will be identical if

$$\frac{f'(\theta)}{f(\theta)} = \csc \theta. \tag{8}$$

After integrating, we find that  $f(\theta)$  = C tan  $\theta/2$ , where C is an arbitrary constant of integration which we may set equal to unity. This then is the required mapping function:

Due to the judicious choice of C, this function will map the surface of any hemisphere with center at the origin into the interior of the unit circle in the  $\rho$ ,  $\phi_{\rm C}$  plane. Figure 3 shows the mapping of the boundaries of our problem. It can be seen that the radial lines  $\phi_{\rm C}$  = constant describe the  $\phi$  angle of the spherical coordinates, and the concentric circles  $\rho$  = tan  $\theta/2$  correspond to the  $\theta$  angle of the spherical system.

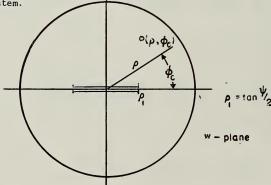


Figure 3 Fin Configuration in the Complex w-Plane

It has been shown that this function 9 effects the required mapping and also preserves Laplace's equation. Therefore any solution of Laplace's equation in the w-plane is also a solution in the hemispherical surface.

All that remains is to find a solution for the cylindrical problem in the w-plane.

#### 3. AN EXAMPLE: THE BICONICAL ANTENNA

It is interesting to note that the transformation from the sphere to the w-plane affords a direct solution for the characteristic impedance of a biconical antenna. The configuration in the w-plane is two concentric circles as shown in Figure 4. This is the familiar coaxial line problem for which the solution is known.<sup>2</sup>

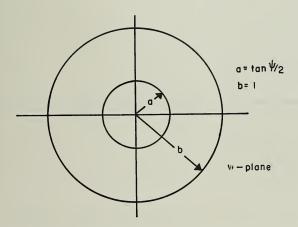


Figure 4 Cone Configuration in the Complex w-Plane

$$Z_0 = \frac{\eta}{2\pi} \log b/a$$

where  $\eta$  is the intrinsic impedance of the medium between the conductors. Substituting for b and a,

$$Z_o = \frac{n}{2\pi} \log \left( \frac{1}{\tan \psi/2} \right)$$

<sup>2.</sup> Ramo and Whinnery, "Fields and Waves in Modern Radio", Wiley, 1953, p. 119.

where  $\psi/2$  is the half angle of the cone. Therefore, for a cone above its image plane,

$$Z_0 = (\eta/2\pi) \log \cot \psi/2. \tag{10}$$

This is the exact solution given by Schelkunoff and Friis.

#### 4'. SOLUTION OF THE PROBLEM

Returning to the original problem, we find that the solution can be written in terms of elliptic functions.<sup>3</sup> First consider the mapping which transforms the w-plane into the z-plane through the use of the intermediate  $\sigma$ -plane. See Figure 5.

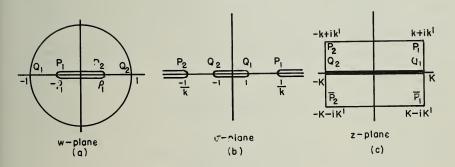


Figure 5 Maps of the w-, o-, and z-Planes

$$\sigma = -1/2 [w + (1/w)]$$
 (11)

$$z = \int_0^\sigma \frac{d\sigma}{\sqrt{(1-\sigma^2)(1-k^2\sigma^2)}}$$
 (12)

$$\frac{1}{k} = \frac{1}{2} \left( \rho_{1} + \frac{1}{\rho_{1}} \right). \tag{13}$$

<sup>3,</sup> F. Oberhettinger and W. Magnus, "Anwendung der Elliptischen Funkionen in Physik und Technik," Springer, 1949, pp. 50-65.

Equation 11 maps the w-plane, including the unit circle and its internal slit, into the whole  $\sigma$ -plane furnished with the slits as indicated in Figure 5(b). Note the unusual correspondence between the points  $P_1$ ,  $P_2$ ,  $Q_1$ , and  $Q_2$ . The upper half  $\sigma$ -plane may be mapped onto the rectangle  $Q_1Q_2P_2P_1$  in the z-plane by the use of the Schwarz-Christoffel transformation Equation 12. Due to the symmetry principle of Riemann and Schwarz, the lower half  $\sigma$ -plane is mapped onto a rectangle in the z-plane which is formed by inverting the rectangle  $Q_1Q_2P_2P_1$  about the line  $Q_1Q_2$ . The relation between the modulus k and  $\rho$  is given by Equation 13.

With the help of Equation 12 both K and K' can be expressed in terms of k. We have

$$K = \int_{0}^{1} \frac{dt}{\sqrt{(1-x^{2})(1-k^{2}t^{2})}}, \qquad (14)$$

$$iK' = \int_{1}^{1/k} \frac{ds}{\sqrt{(1-s^{2})(1-k^{2}s^{2})}}.$$

and

The latter integral can be brought into a more elegant form. It we make the substitution

 $s = (1-k'^2t^2)^{-1/2} ,$  where  $k'^2 + k^2 = 1, \qquad (15)$ 

we obtain

$$K' = \int_{0}^{1} \frac{dt}{\sqrt{(1-t^{2})(1-k'^{2}t^{2})}}.$$
 (16.)

Notice that Equations 14 and 16 are complete elliptic integrals of the first kind of modulus k and k', respectively.

The configuration in the z-plane can be considered as the parallel combination of two parallel plate transmission lines. The parallel

plate line has a characteristic impedance given by

$$Z_0 = \eta d/b$$
,

where η is the intrinsic impedance of the medium between the conductors, d is the distance between the plates, and b is the width of the plates. Note that this is the exact solution; there are no fringing effects because the total electric field in the σ-plane is mapped into the interior of the two period rectangles in the z-plane. Thus the characteristic impedance of the parallel combination of these transmission lines is given by

$$Z_o = \frac{1}{2} \eta \frac{K'}{2K} = 30\pi \frac{K'}{K}$$
 (17)

From Equation 13,

$$k = \frac{2\rho_1}{\rho_1^2 + 1} .$$

Upon substituting  $\rho_1$  = tan  $\psi/2$  (from Figure 3), it is seen that

$$k = \sin \psi$$
 $k' = \cos \psi$  (18)

The solution of the problem is now complete; see Figure 6 for a graph of the characteristic impedance versus the half angle  $\psi$  of the antenna.

Table 1. CHARACTERISTIC IMPEDANCE OF THE FIN AND CONE.

Ψο	Z <sub>o</sub> - FIN	Z <sub>o</sub> CONE
0	. <b>co</b>	ω
5	229	188
10	187	146
15	163	122
20	146	104
25	132	90.5
30	121	79.0
35	111	69.3
40	102	60.6
45	94.2	52.8
50	87.0	45.8
55	80.3	39.2
6.0	73.7	33.0
65	67.3	27.0
70	61.0	21.4
75	54'.4'	15.9
80	47.3	10.5
85	38.6	5.3
90	0	0

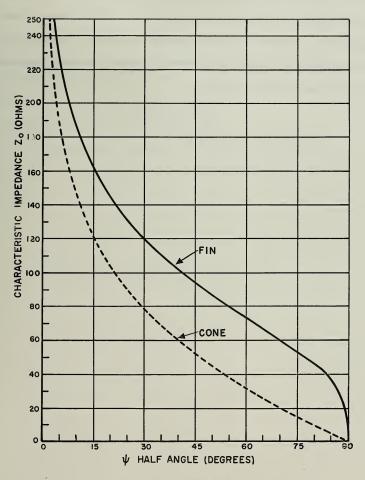


Figure 6 Characteristic Impedance of the Fin and the Cone Above a Ground Plane

#### 5. CONCLUDING REMARKS

It should be pointed out that the method here employed to solve the fin antenna is quite general. As outlined in Section 2, this method may be applied to any uniform structure in which  $\theta$  = f ( $\phi$ ). However, the mapping of the w-plane into a more amenable geometry, as in Section 4, may be exceedingly difficult. Furthermore the mapping will, in general, differ from one problem to another.

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#### APPENDIX A

It will be demonstrated that, under certain conditions, Maxwell's equations reduce to Laplace's equation in two dimensions, and that the boundary conditions on the surface of the conductor reduce to the boundary conditions of electrostatics.

Maxwell's equations for a lossless medium can be written as:

$$jωε\underline{E} = \nabla \times \underline{H}$$

$$-jωμ\underline{H} = \nabla \times E,$$
(19)

Let us determine if Maxwell's equations have a solution given in terms of a scalar function of three spherical coordinates. Consider

$$\mathbf{H} = \nabla \times \mathbf{r} \ \Pi, \tag{20}$$

where  $\underline{r}$  is the unit radial vector and  $\Pi$  is a scalar function. I Equation 20 is a solution, it must satisfy the vector wave equation

$$\nabla \times \nabla \times \underline{\mathbf{H}} - \beta^2 \underline{\mathbf{H}} = 0. \tag{21}$$

Substituting 20 in 21 we find that

$$\nabla \times (\nabla \times \nabla \times \mathbf{r} \Pi - \beta^2 \mathbf{r} \Pi) = 0. \tag{22}$$

Equation 22 will hold if

$$\nabla \times \nabla \times \mathbf{r} \Pi - \beta^2 \mathbf{r} \Pi = - \nabla (j\omega eV)$$
 (23)

where V is any scalar function. (The reason for the use of the arbitrary constant multiplier will become apparent later.)

Equating the r,  $\theta$ , and  $\phi$  components of each side of Equation 23, we find that

$$-j\omega\varepsilon \frac{\partial V}{\partial \mathbf{r}} + \beta^2\Pi + \frac{1}{\mathfrak{r}^2\sin^2\theta} \left[ \sin\theta \frac{\partial}{\partial\theta} (\sin\theta \frac{\partial\Pi}{\partial\theta}) + \frac{\partial^2\Pi}{\partial\phi^2} \right] = 0,$$
(24)

$$\frac{\partial^2 \Pi}{\partial r \partial \phi} = -j \omega \varepsilon \frac{\partial V}{\partial \phi}, \qquad (25)$$

$$\frac{\partial^2 \Pi}{\partial \mathbf{r} \partial \theta} = -\mathbf{j} \omega \epsilon \frac{\partial \mathbf{V}}{\partial \theta} . \tag{26}$$

Equations 25 and 26 will be satisfied if

$$-j\omega \varepsilon V' = \frac{\partial \Pi}{\partial \mathbf{r}}.$$
 (27)

Substituting for V in Equation 24, we find that, for Equation 20 to be a solution of Maxwell's equations,  $\Pi$  must satisfy this equation:

$$\frac{\partial^{2}\Pi}{\partial \mathbf{r}^{2}} + \beta^{2}\Pi + \frac{1}{\mathbf{r}^{2}\sin^{2}\theta} \begin{bmatrix} \sin\theta \frac{\partial}{\partial\theta} \left( \sin\theta \frac{\partial\Pi}{\partial\theta} \right) + \frac{\partial^{2}\Pi}{\partial\phi^{2}} \end{bmatrix} = 0.$$
 (28)

If we assume a product solution of the form  $\Pi$  = RT, where R is a function of r alone and T is a function of  $\theta$  and  $\phi$ , we find that

$$\frac{\mathbf{r}^2}{R} \left( \frac{\partial^2 \mathbf{R}}{\partial \mathbf{r}^2} + \beta^2 \mathbf{R} \right) + \frac{1}{T \sin^2 \theta} \left[ \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \mathbf{T}}{\partial \theta} \right) + \frac{\partial^2 \mathbf{T}}{\partial \phi^2} \right] = 0. \tag{29}$$

Since Equation 29 is in the separated form, both parts can be set equal to a constant. It is obvious that one solution of this equation can be found when

$$\frac{\partial^2 R}{\partial r^2} + \beta^2 R = 0 \tag{30}$$

and

$$\sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \mathbf{T}}{\partial \theta} \right) + \frac{\partial^2 \mathbf{T}}{\partial \phi^2} = 0.$$
 (31)

We shall return to Equations 30 and 31 presently, but first let us examine the class of problems which can be handled by this restricted solution. Equation 20 limits the solution of Maxwell's equations to TM modes, that is, H =0. If we substitute 20 in Maxwell's equations and solve for E we find that, after some algebraic manipulation,

$$j\omega\epsilon \ \underline{E} = \underline{r} \left\{ \frac{1}{r^2 \sin^2 \theta} \left[ \sin \theta \ \frac{\partial}{\partial \theta} (\sin \theta \ \frac{\partial \Pi}{\partial \theta}) + \frac{\partial^2 \Pi}{\partial \phi^2} \right] \right\}$$

$$+ \underline{\theta} \left\{ \underline{1} \ \frac{\partial^2 \Pi}{\partial r \partial \theta} \right\}$$

$$+ \underline{\phi} \left\{ \frac{1}{r} \ \frac{\partial^2 \Pi}{\partial r \partial \phi} \right\},$$
(32)

where  $\underline{r},\underline{\theta}$ , and  $\underline{\phi}$  are the unit vectors in the r,  $\theta$ ,  $\phi$  directions, respectively. Notice that  $E_r$  also vanishes due to the assumption of Equation 31. Using Equation 27, we may now write

$$\underline{\mathbf{E}} = -\nabla_{\mathbf{r}} \mathbf{V}', \tag{33}$$

where  $\nabla_{\bf t}$  is the transverse gradient operator. Thus we see that the assumptions which have been made are justifiable, and lead to solutions for  $\underline{E}$  and  $\underline{H}$  such that  $\underline{E}_{\bf r}$  =  $\underline{H}_{\bf r}$  = 0. This is the familiar principle or TEM mode for spherical waves.

If we solve the differential equation 30 we find that

$$R = e^{\pm j\beta r}.$$
 (34)

We choose to use the minus sign only, since propagation is outward from the origin. Hence  $\Pi$  =  $e^{-j}\beta r$  can be substituted in Equation 28, which becomes

$$\sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) + \frac{\partial^2 T}{\partial \phi^2} = 0.$$
 (35)

This is Laplaces equation with  $\partial T/\partial r$  = 0. Note that  $\Pi$  and V also satisfy this equation.

Let us now examine how the boundary conditions on  $\underline{H}$  and  $\underline{E}$  restrict  $\Pi$  and V. At the surface of the conductors  $\underline{E} \times \underline{N} = 0$ . ( $\underline{N}$  is the outward directed normal). Since  $\underline{E} = -\nabla_t V$ ,  $-\nabla_t V \times \underline{N} = 0$ . This shows that V and  $\partial \Pi / \partial r$  must be constant on the surface of each conductor. These are the boundary conditions of electrostatics. Thus we have shown that our choice of auxiliary functions satisfies Maxwell's equations and the boundary conditions, and leads to a solution which must satisfy Laplace's equation.



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